NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 3 Statistics and Modelling, 2007

90642 Calculate confidence intervals for population parameters

Credits: Three
9.30 am Thursday 29 November 2007

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–STATF.

You should answer ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	Achievement Criteria	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Calculate confidence intervals for population parameters.	Demonstrate an understanding of confidence intervals.	Demonstrate an understanding of the theory behind confidence intervals.
Overall Level of Performance		

You are advised to spend 35 minutes answering the questions in this booklet.

THE JUICE FACTORY

The Juice Factory has machines that dispense different flavoured fruit juices into bottles.

Different aspects of the juice production process are checked regularly.

QUESTION ONE

A random sample of 40 bottles of lemon juice was taken and the volume of juice in each bottle was measured. The sample mean was 752.7 mL and the sample standard deviation was 6.3 mL.

Find a 99% confidence interval for the mean volume of juice in all bottles of lemon juice that are produced by the lemon juice machine at the factory.

QUESTION TWO

The company wants to compare the output from the lemon juice machine with that from the orange juice machine.
As well as using the lemon juice data in Question One, data was obtained from a random sample of 40 orange juice bottles.

The table summarises the data that was collected:

	Lemon	Orange
Sample size (number of bottles)	40	40
Sample mean (mL of juice)	752.7	748.1
Sample standard deviation (mL of juice)	6.3	8.2

Let μ_1 be the mean volume that the lemon juice machine dispenses into bottles and let μ_2 be the mean volume that the orange juice machine dispenses into bottles.

Find a 95% confidence interval for $\mu_1 - \mu_2$.

QUESTION THREE

The Juice Factory fills recycled bottles with its juice. A random sample of 120 bottles was checked to see if they were clean enough to use.

The manager reported that 5% of the 120 bottles were rejected because they were not clean enough.

(a)   Find a 90% confidence interval for the proportion of all bottles that were rejected because they were not clean enough.

_____

_____

_____

(b)   Explain, in terms of the proportion of all recycled bottles used by The Juice Factory, the meaning of the confidence interval in Question Three (a).

_____

_____

_____

_____

_____

_____

_____

_____

QUESTION FOUR

A customer claims that the mean volume of juice in bottles of blackcurrant juice is different from the 750 mL that is stated on them.

A random sample of 50 bottles of blackcurrant juice was taken and the volume of juice in each bottle was measured. The sample mean was 747.2 mL and the sample standard deviation was 9.4 mL.

Is the customer's claim justified? Use a 95% confidence interval to support your answer.

_____

_____

_____

_____

_____

_____

QUESTION FIVE

The factory manager decided to check the results in Question Four by taking a second sample of blackcurrant juice bottles.

She wanted the **total width** of a 95% confidence interval for the mean volume of juice in all bottles of blackcurrant juice to be no more than 3.0 mL.

Find the minimum sample size that is required to meet this condition.

(Assume that the standard deviation for the amount of juice dispensed into bottles by the blackcurrant juice machine is 9.4 mL.)

_____

_____

_____

_____

_____

_____

_____

QUESTION SIX

Random samples of size n are taken from a population X with mean μ and standard deviation σ.

Use expectation algebra to prove that the standard deviation for the distribution of sample means, $\bar{X}$, is $\dfrac{\sigma}{\sqrt{n}}$.

Note: $\bar{X} = \dfrac{X_1 + X_2 + ... + X_n}{n}$

QUESTION SEVEN

A major supermarket regularly receives batches of bottles of lemon juice from The Juice Factory.

The supermarket will reject an entire batch if a random sample of 30 bottles of lemon juice produces a mean volume of juice that is less than 750.0 mL.

Suppose that the volume of lemon juice dispensed in all bottles in one batch has a mean of 752.5 mL and standard deviation of 5.1 mL. Calculate the probability that an entire batch will be rejected.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

QUESTION EIGHT

A random sample of size n is taken from a population having a known standard deviation σ. A 95% confidence interval for the population mean is calculated using the sample mean.

A second random sample of size $2n$ is taken from the same population and a 95% confidence interval for the population mean is calculated using its sample mean.

How many times greater is the width of the first confidence interval than the width of the second confidence interval?

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Extra paper for continuation of answers if required.
Clearly number the question.

Question number

90642